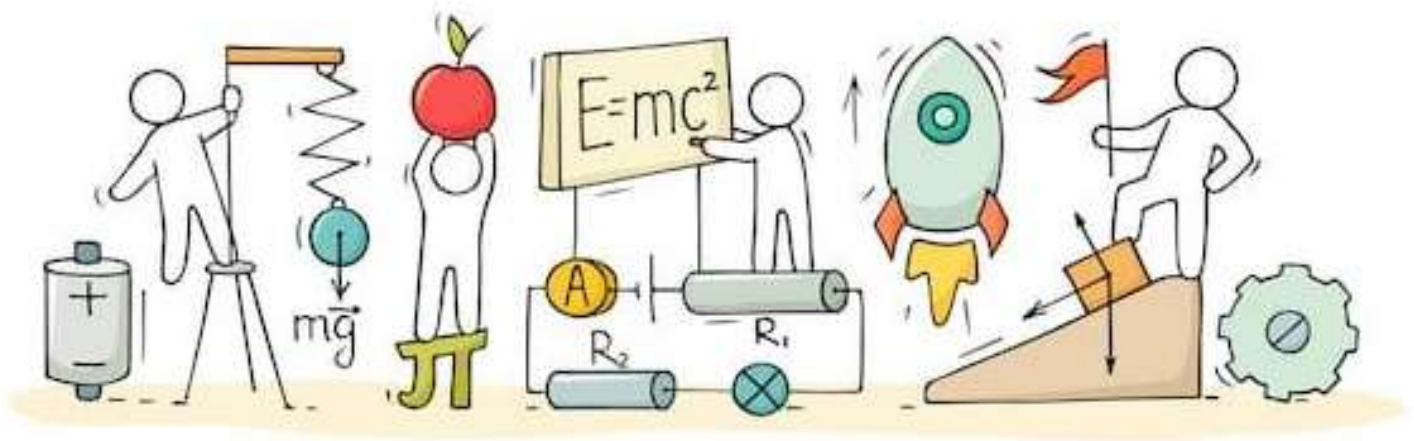


PHYSICS

Chapter 8: Gravitation



Gravitation

Top Formulae

Newton's law of gravitation	$F = \frac{Gm_1m_2}{r^2},$ $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$
Acceleration due to gravity	$g = \frac{GM}{R^2} = \frac{4}{3} \pi GR\rho$
Variation of g	(a) Altitude (height) effect $g' = g \left(1 + \frac{h}{R} \right)^{-2}$ If $h \ll R$, then $g' = g \left(1 - \frac{2h}{R} \right)$ (b) Effect of depth $g'' = \left(1 - \frac{d}{R} \right)$
Intensity of gravitational field	$\vec{E}_g = \frac{GM}{r^2} (-\vec{r})$ For the Earth, $E_g = g = 9.86 \text{ m/s}^2$
Gravitational potential	$v_g = -\int_{\infty}^r \vec{E} \cdot d\vec{r}$ For points outside ($r > R$), $v_g = -\frac{GM}{r}$ For points inside it, $r < R$ $v_g = -GM \left[\frac{3R^2 - r^2}{2R^3} \right]$
Change in potential energy in going to a height h above the surface	$\Delta U_g = mgh \quad \text{if } h \ll R_e$ In general, $\Delta U_g = \frac{mgh}{\left(1 + \frac{h}{R} \right)}$

Orbital velocity of a satellite	$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$ $v_0 = \sqrt{\frac{GM}{R+h}} \quad \because r = h + R$ $\text{If } h \ll R \quad v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR} = 8 \text{ km/sec.}$
Velocity of projection	Loss of KE = Gain in PE $\frac{1}{2}mv_p^2 = -\frac{gMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$ $v_p = \left[\frac{2GMh}{R(R+h)}\right]^{1/2} = \left[\frac{2gh}{1+\frac{h}{R}}\right]^{1/2} \quad (\because GM = gR^2)$
Period of revolution	$T = \frac{2\pi r}{v_0} = \frac{2\pi(R+h)^{3/2}}{R\sqrt{g}}$ Or $T^2 = \frac{4\pi^2 r^3}{GM}$ $\text{If } h \ll R \quad T = \frac{2\pi R^{3/2}}{R\sqrt{g}} = 1\frac{1}{2} \text{ hr}$
Kinetic energy of a satellite	$KE = \frac{GMm}{2r} = \frac{1}{2}mv^2$
Potential energy of a satellite	$U = -\frac{GMm}{r}$
Binding energy of a satellite	$= \frac{1}{2} \frac{GMm}{R}$
Escape velocity	$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = R\sqrt{\frac{8\pi Gd}{3}}$ $v_e = v_0 \sqrt{2}$
Effective weight in a satellite	$w = 0$ Satellite behaves like a freely falling body.
Kepler's laws for planetary motion	(a) Elliptical orbit with the Sun at one focus (b) Areal velocity constant $dA/dt = \text{constant}$ (c) $T^2 \propto r^3$; $r = (r_1 + r_2)/2$

Top Concepts

- Newton's law of universal gravitation: The gravitational force of attraction between any two particles of masses m_1 and m_2 separated by a distance r has the magnitude.

$$F = G \frac{m_1 m_2}{r^2}$$

where G , the universal gravitational constant, has the value $6.672 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$.

- From the principle of superposition, each force acts independently and uninfluenced by the other bodies. The resultant force F_R is then found by vector addition.

$$F_R = F_1 + F_2 + \dots + F_n = \sum_{i=1}^n F_i$$

- Acceleration due to gravity $g = \frac{GM}{r^2}$
- For small heights h above the Earth's surface, the value of g decreases by a factor $(1 - 2h/R)$.
- The gravitational potential energy of two masses separated by a distance r is inversely proportional to r .
- The potential energy is never positive; it is zero only when the two bodies are infinitely far apart.
- The gravitational potential energy associated with two particles separated by a distance r is given by

$$V = -\frac{Gm_1 m_2}{r}$$

where V is taken to be zero at $r \rightarrow \infty$

- The total mechanical energy is the sum of the kinetic and potential energies. The total energy is a constant of motion.
- If an isolated system consists of a particle of mass m moving with a speed v in the vicinity of a massive body of mass M , then the total mechanical energy of the particle is given by

$$E = \frac{1}{2} m v^2 - \frac{GMm}{r}$$

If m moves in circular orbit of radius ' a ' about M , where $M \gg m$, then the total energy of the system is

$$E = -\frac{GMm}{2a}$$

- The escape speed from the surface of the Earth is

$$v_e = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E}$$

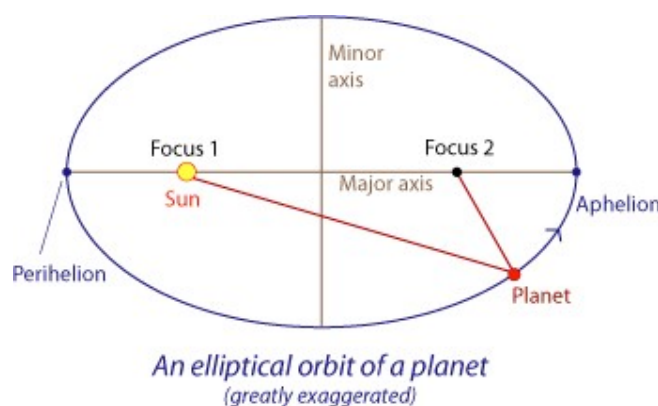
and has a value of 11.2 km s^{-1} .

- Kepler's laws of planetary motion:**

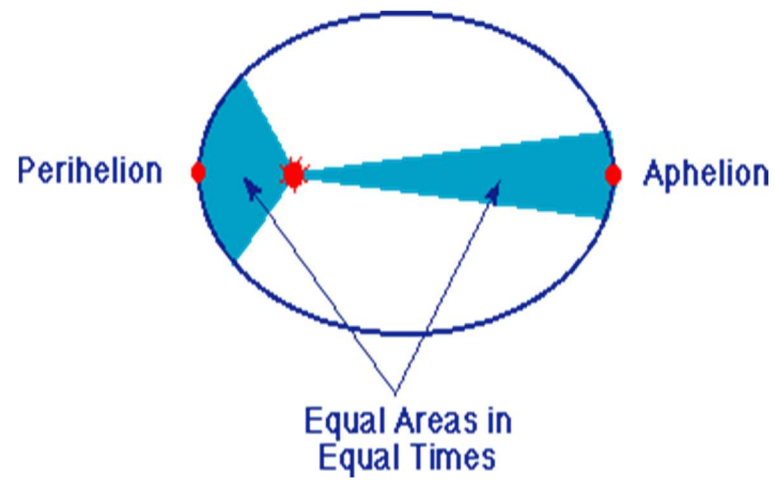
- Law of Orbits:** The orbit of the planet is elliptical with the Sun at one of the focus.
 - Law of Areas:** The line joining the planet to the Sun sweeps out equal area in equal interval of time.
 - The square of the planet's time period of revolution T is proportional to the cube of the semi-major axis a .
- A geostationary satellite moves in a circular orbit in the equatorial plane at an approximate distance of 36,000 km.

Diagrams

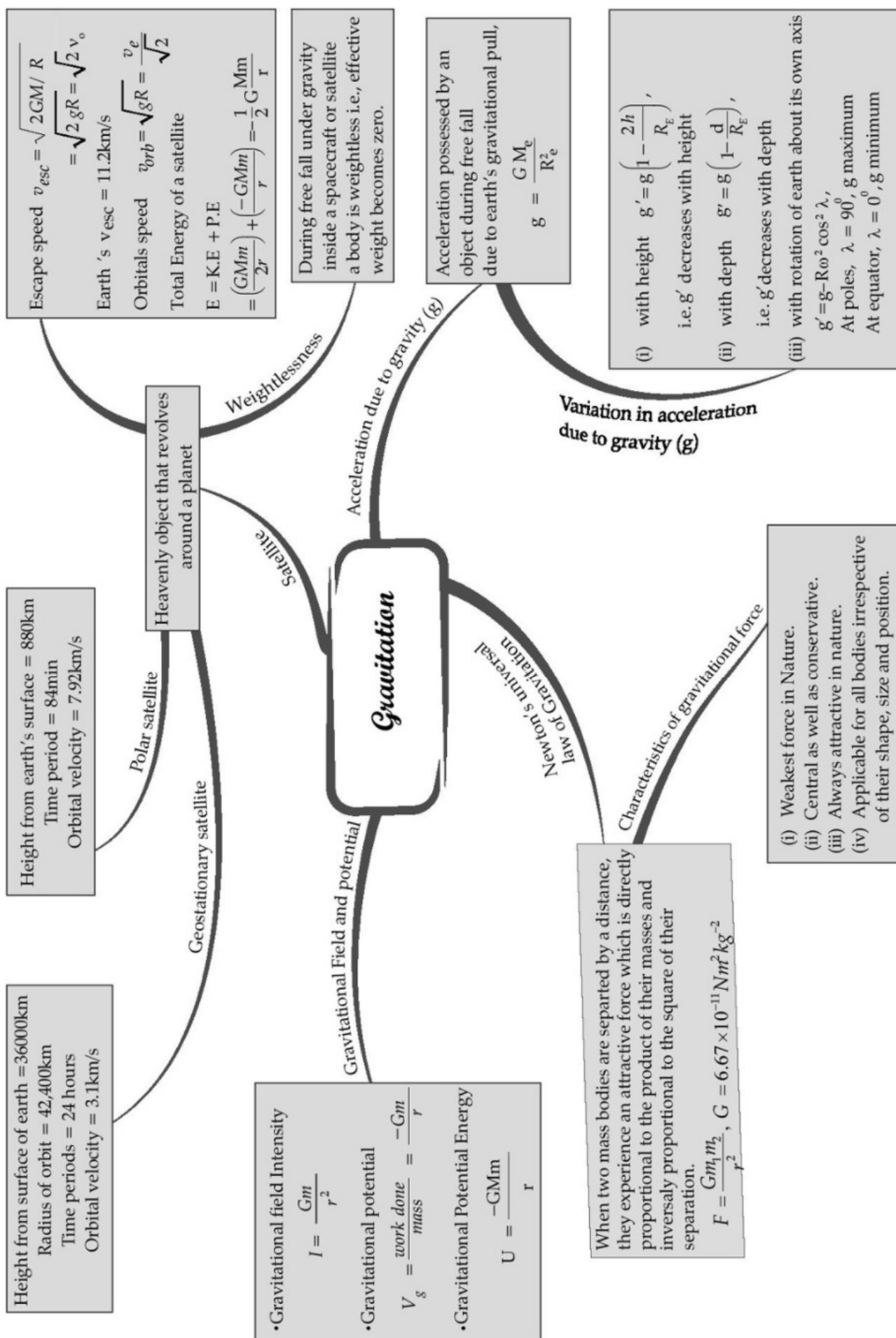
Kepler's first law



Kepler's second law



CHAPTER - 8 : GRAVITATION



Important Questions

Multiple Choice questions-

1. A body is projected vertically from the surface of the earth of radius R with velocity equal to half of the escape velocity. The maximum height reached by the body is
 - (a) R
 - (b) $R/2$
 - (c) $R/3$
 - (d) $R/4$
2. When the planet comes nearer the sun moves
 - (a) fast
 - (b) slow
 - (c) constant at every point
 - (d) none of the above
3. Keplers second law regarding constancy of arial velocity of a planet is a consequence of the law of conservation of
 - (a) energy
 - (b) angular momentum
 - (c) linear momentum
 - (d) none of these
4. The escape velocity for a body projected vertically upwards from the surface of the earth is 11km/s . If the body is projected at an angle of 45° with the vertical, the escape velocity will be
 - (a) $11/\sqrt{2}\text{ km/s}$
 - (b) $11\sqrt{2}\text{ km/s}$
 - (c) 2 km/s
 - (d) 11 km/s
5. The radii of the earth and the moon are in the ratio $10 : 1$ while acceleration due to gravity on the earths surface and moons surface are in the ratio $6 : 1$. The ratio of escape velocities from earths surface to that of moon surface is
 - (a) $10 : 1$
 - (b) $6 : 1$

(c) 1.66 : 1

(d) 7.74 : 1

6. The escape velocity of a body from the surface of the earth is v . It is given a velocity twice this velocity on the surface of the earth. What will be its velocity at infinity?

(a) v

(b) $2v$

(c) $\sqrt{2}v$

(d) $\sqrt{3}v$

7. The period of geostationary artificial satellite is

(a) 24 hours

(b) 6 hours

(c) 12 hours

(d) 48 hours

8. If the radius of the earth were to shrink by 1% its mass remaining the same, the acceleration due to gravity on the earth's surface would

(a) decrease by 2%

(b) remain unchanged

(c) increase by 2%

(d) will increase by 9.8%

9. The mean radius of the earth is R , its angular speed on its own axis is ω and the acceleration due to gravity at earth's surface is g . The cube of the radius of the orbit of a geo-stationary satellite will be

(a) r^2g / ω

(b) $R^2\omega^2 / g$

(c) $RG \omega^2$

(d) R^2g / ω^2

10. If escape velocity from the earth's surface is 11.2 km/sec. then escape velocity from a planet of mass same as that of earth but radius one fourth as that of earth is

(a) 11.2 km/sec

(b) 22.4 km/sec

(c) 5.65 km/sec

(d) 44.8 km/sec

Very Short:

1. What velocity will you give to a donkey and what velocity to a monkey so that both escape the gravitational field of Earth?
2. How does Earth retain most of the atmosphere?
3. Earth is continuously pulling the moon towards its center. Why does not then, the moon falls on the Earth?
4. Which is greater out of the following:
 - (a) The attraction of Earth for 5 kg of copper.
 - (b) The attraction of 5 kg copper for Earth?
5. Where does a body weigh more – at the surface of Earth or in a mine?
6. How is it that we learn more about the shape of Earth by studying the motion of an artificial satellite than by studying the motion of the moon?
7. If the Earth is regarded as a hollow sphere, then what is the weight of an object below the surface of Earth?
8. What is the formula for escape velocity in terms of g and R ?
9. What is the orbital period of revolution of an artificial satellite revolving in a geostationary orbit?
10. Can we determine the mass of a satellite by measuring its time period?

Short Questions:

1. Explain how the weight of the body varies en route from the Earth to the moon. Would its mass change?
2. Among the known type of forces in nature, the gravitational force is the weakest. Why then does it play a dominant role in the motion of bodies on the terrestrial, astronomical, and cosmological scale?
3. Show that the average life span of humans on a planet in terms of its natural years is 25 planet years if the average span of life on Earth is taken to be 70 years.
4. Hydrogen escapes faster from the Earth than oxygen. Why?
5. In a spaceship moving in a gravity-free region, the astronaut will not be able to distinguish between up and down. Explain why?
6. Why the space rockets are generally launched from west to east?
7. Explain why the weight of a body becomes zero at the centre of Earth.
8. We cannot move even our little fingers without disturbing the whole universe. Explain why.

Long Questions:

1. (a) Derive the expression for the orbital velocity of an artificial Earth's satellite. Also, derive its value for an orbit near Earth's surface.
(b) Derive the expression for escape velocity of a body from the surface of Earth and show that it $\sqrt{2}$ times the orbital velocity close to the surface of the Earth. Derive its value for Earth.
2. (a) Explain Newton's law of gravitation.
(b) Define gravitational field intensity. Derive its expression at a point at a distance x from the center of Earth. How is it related to acceleration due to gravity?
3. Discuss the variation of acceleration due to gravity with:
 - (a) Altitude or height
 - (b) Depth
 - (c) Latitude i.e. due to rotation of Earth.

Assertion Reason Questions:

1. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
 - (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
 - (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
 - (c) Assertion is correct, reason is incorrect
 - (d) Assertion is incorrect, reason is correct.

Assertion: Gravitational potential of earth at every place on it is negative.

Reason: Everybody on earth is bound by the attraction of earth.

2. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
 - (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
 - (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
 - (c) Assertion is correct, reason is incorrect
 - (d) Assertion is incorrect, reason is correct.

Assertion: Planets appear to move slower when they are farther from the sun than when they are nearer.

Reason: All planets move in elliptical orbits with sun at one of the foci of the ellipse.

✓ **Answer Key:**

Multiple Choice Answers-

1. Answer: (c) R/3

2. Answer: (a) fast
3. Answer: (b) angular momentum
4. Answer: (d) 11 km/s
5. Answer: (d) 7.74 : 1
6. Answer: (d) $\sqrt{3}v$
7. Answer: (a) 24 hours
8. Answer: (c) increase by 2%
9. Answer: (d) R^2g / w^2
10. Answer: (b) 22.4 km/sec

Very Short Answers:

1. Answer: We will give them the same velocity as escape velocity is independent of the mass of the body.
2. Answer: Due to force of gravity.
3. Answer: The gravitational force between the Earth and the moon provides the necessary centripetal force to the moon to move around the Earth. This centripetal force avoids the moon to fall onto the Earth.
4. Answer: Same.
5. Answer: At the surface of Earth, a body weighs more.
6. Answer: This is because an artificial satellite is closer to the Earth than Moon.
7. Answer: Zero.
8. Answer: $V_e = \sqrt{2gR}$.
9. Answer: It is 24 hours.
10. Answer: Yes.

Short Questions Answers:

1. Answer: When a body is taken from Earth to the moon, then its weight slowly decreases to zero and then increases till it becomes $\frac{1}{6}$ th of the weight of the body on the surface of the moon.

We know that $mgh = mg \left(1 - \frac{2h}{R}\right)$

As h increases, gh , and hence mgh , decreases. When $R = \frac{R}{2}$ the force of attraction of Earth is equal to the force of attraction of the moon.

Then $gh = 0$, so mg becomes zero, and the value of g on the moon's surface is $\frac{1}{6}$ th of its value on the surface of Earth. Hence on increasing h beyond $\frac{R}{2}$, mg starts increasing due to the gravity of the moon. Its mass remains constant.

2. Answer: Electrical forces are stronger than gravitational forces for a given distance, but they can be attractive as well as repulsive, unlike gravitational force which is always attractive. As a consequence, the forces between massive neutral bodies are predominantly gravitational and hence play a dominant role at long distances. The strong nuclear forces dominate only over a range of distances of the order of 10^{-14} m to 10^{-15} m.
3. Answer: Take the distance between Earth and Sun twice the distance between Earth and planet. According to Kepler's third law of planetary motion,

$$\left(\frac{T_e}{T_p}\right)^2 = \left(\frac{R_e}{R_p}\right)^3$$

where T_e , T_p is the average life span on Earth and planet respectively.

R_e = distance between Earth and Sun.

R_p = distance between Earth and planet.

Here, $R_e = 2R_p$

$$\begin{aligned} \text{or} \quad \frac{R_e}{R_p} &= 2; \\ T_e &= 70 \text{ years} \\ T_p &= ? \end{aligned}$$

$$\therefore \left(\frac{T_e}{T_p}\right)^2 = (2)^3$$

$$\text{or} \quad \frac{T_e}{T_p} = (2)^{\frac{3}{2}}$$

$$\begin{aligned} \text{or} \quad T_p &= \frac{T_e}{(2)^{\frac{3}{2}}} = \frac{70}{\sqrt{2^3}} = \frac{70}{\sqrt{8}} = \frac{70}{2\sqrt{2}} \\ &= 35 \times \frac{\sqrt{2}}{2} = 17.5 \times 1.414 \\ &= 24.75 \approx 25 \text{ planet years.} \end{aligned}$$

4. Answer: The thermal speed of hydrogen is much larger than oxygen. Therefore a large number of hydrogen molecules are able to acquire escape velocity than that of oxygen molecules. Hence hydrogen escapes faster from the Earth than oxygen.

5. Answer: The upward and downward sense is due to the gravitational force of attraction between the body and the earth. In a spaceship, the gravitational force is counterbalanced by the centripetal force needed by the satellite to move around the Earth in a circular orbit. Hence in the absence of zero force, the astronaut will not be able to distinguish between up and down.
6. Answer: Since the Earth revolves from west to east around the Sun, so when the rocket is launched from west to east, the relative velocity of the rocket = launching velocity of rocket + linear velocity of Earth. Thus the velocity of the rocket increases which helps it to rise without much consumption of the fuel. Also, the linear velocity of Earth is maximum in the equatorial plane.
7. Answer: We know that the weight of a body at a place below Earth's surface is given by $W = mg_d$ (i)

Where g_d = acceleration due to gravity at a place at a depth 'd' below Earth's surface and is given

$$g_d = g \left(1 - \frac{d}{R} \right) \quad \dots (ii)$$

At the centre of Earth, $d = R$,

$$\begin{aligned} \therefore g_d &= g \left(1 - \frac{R}{R} \right) \\ &= g(1 - 1) = g \times 0 = 0 \end{aligned}$$

From Eqn. (i) $W = 0$ at the center of Earth.

i.e., g decreased with depth and hence becomes zero at the center of Earth, so $W = 0$ at Earth's center.

8. Answer: According to Newton's law of gravitation, every particle of this universe attracts every other particle with a force that is inversely proportional to the square of the distance between them. When we move our fingers, the distance between the particle's changes, and hence the force of attraction changes which in turn disturbs the whole universe.

Long Questions Answers:

1. Answer:

1. Let m = mass of the satellite.

M, R = mass and radius of Earth.

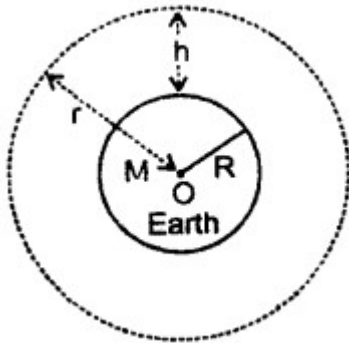
h = height of the satellite above the surface of Earth.

r = radius of the orbit of the satellite

$= R + h$.

v_0 = orbital velocity of the satellite.

The centripetal force $\frac{mv_0^2}{r}$ required by the satellite to move in a circular orbit is proved by the gravitational force between satellite and the Earth.



$$\text{i.e.} \quad \frac{mv_0^2}{r} = \frac{GMm}{r^2}$$

$$\begin{aligned} \text{or} \quad v_0 &= \sqrt{\frac{GM}{r}} \\ &= \sqrt{\frac{GM}{R+h}} \quad \dots (i) \end{aligned}$$

Also we know that

$$g = \frac{GM}{R^2}$$

$$\text{or} \quad GM = gR^2$$

$$\begin{aligned} \therefore v_0 &= R \sqrt{\frac{g}{R+h}} \\ &= R \sqrt{\frac{g}{(R+h)}} \quad \dots (ii) \end{aligned}$$

$$\text{Also} \quad g_h = \frac{GM}{(R+h)^2} \quad \dots (iii)$$

from (i) and (iii), we get

$$v_0 = \sqrt{g_h(R+h)}$$

If the satellite is close to the earth's surface, then $h \approx 0$

$$\therefore \text{from (ii), } v_0 = \sqrt{gR}$$

$$\text{Putting, } g = 9.8 \text{ ms}^{-2}, R = 6.38 \times 10^6 \text{ m}$$

$$\therefore v_0 = \sqrt{9.8 \times 6.38 \times 10^6} = 7.9 \text{ kms}^{-1}.$$

$$\begin{aligned} \therefore v_0 &= \sqrt{gR} = R\sqrt{\frac{g}{R+h}} \\ &= \sqrt{\frac{GM}{R+h}} = \sqrt{g_h(R+h)} \end{aligned}$$

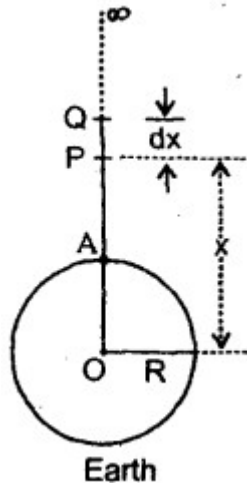
2. Escape velocity is the minimum velocity with which a body is projected from Earth's surface so as to just escape its gravitational pull or of any other planet. It is denoted by v_e .

Expression: Consider the earth to be a homogenous sphere of radius R , mass M , center O , and density ρ .

Let m = mass of the body projected from point A on the surface of Earth with vel. v_e .

$$\therefore \text{K.E. of the body at point A} = \frac{1}{2}mv_e^2 \dots (i)$$

Let it reaches a point P at a distance x from O . If F be the gravitational force of attraction on the body at P , then



$$F = \frac{GMm}{x^2} \dots (ii)$$

Let it further moves to Q by a distance dx .

If dW be the work done in moving from P to Q , then

$$dW = Fdx = \frac{GMm}{x^2} dx \dots (iii)$$

If w be the total work done in moving the body from A to ∞ ,

Then

$$\begin{aligned}
 W &= \int_A^{\infty} dW = \int_A^{\infty} \frac{GMm}{x^2} dx \\
 &= GMm \int_A^{\infty} x^{-2} dx = GMm \left[-\frac{1}{x} \right]_R^{\infty} \\
 &= -GMm \left[\frac{1}{\infty} - \frac{1}{R} \right] \\
 &= -GMm \left(0 - \frac{1}{R} \right) \\
 &= \frac{GMm}{R} \quad \dots (iv)
 \end{aligned}$$

$\therefore$ According to the law of conservation of energy

$$\text{K.E.} = \text{RE}$$

$$\text{or} \quad \frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

$$\text{or} \quad v_e = \sqrt{\frac{2GM}{R}} \quad \dots (v)$$

$$\text{Also} \quad g = \frac{GM}{R^2}$$

$$\therefore \quad v_e = \sqrt{\frac{2gR^2}{R}}$$

$$\text{or} \quad v_e = \sqrt{2gR} \quad \dots (vi)$$

$$\text{Also} \quad M = \frac{4}{3}\pi R^3 \rho$$

$$\therefore \quad v_e = \sqrt{\frac{2G}{R} \cdot \frac{4}{3}\pi R^3 \rho} = \sqrt{\frac{8}{3}\pi G \rho R^2}$$

$$\begin{aligned} \text{for earth} \quad R &= 6.38 \times 10^6 \text{ m} \\ g &= 9.8 \text{ ms}^{-2} \end{aligned}$$

$$\begin{aligned} \therefore \text{ from (vi), } v_e &= \sqrt{2 \times 9.8 \times 6.38 \times 10^6} \\ &= 11.2 \text{ kms}^{-1} \end{aligned}$$

Relation between v_e and v_o : Also we know that the orbital velocity around Earth close to its surface is given

$$\text{by } v_o = \sqrt{gR}$$

$$\text{and } v_e = \sqrt{2gR} = \sqrt{2} \sqrt{gR}$$

$$= \sqrt{2} v_o$$

Hence proved.

2. Answer:

(a) We know that Newton's law of gravitation is expressed mathematically as:

$$F = \frac{Gm_1m_2}{r^2}$$

$$\text{or in vector form } \mathbf{F} = \frac{Gm_1m_2}{r^3} \hat{r},$$

where $\hat{r}$ = unit vector along F

It was found that law is equally applicable anywhere in the universe between small and big objects like stars and galaxies. The value of G remains the same everywhere. (Some scientists have claimed that as the size of the object under consideration becomes big like a galaxy, the value of G also changes). Hence this law of Newton is also called Newton's universal law of gravitation.

The force of attraction is called the force of gravitation or gravitational force. This force is only attractive and is never repulsive. The force is both ways i.e., particle 1 attracts particle 2 and so does particle 2 attracts particle 1.

$$\text{Hence } F_{12} = -F_{21}.$$

The law is a direct outcome of the study of acceleration of bodies. Newton wondered how Moon revolves around the Earth or other planets revolve around the Sun. His calculations showed that the Moon is accelerated by the same amount as does any other object towards the Earth.

His famous narration of the apple falling from the tree and noticing every other object fall towards Earth led to the announcement of his famous law of gravitation about 50 years later in his book 'Principia'.

Out of the known forces in nature, the Gravitational force is the weakest, yet it is the most apparent one as it acts for long distances and between objects which are visible to us. The law of gravitation has been used to determine the mass of heavenly bodies. It has been used to study the atmosphere of planets. Man-made satellites remain in the orbits due to gravitation.

- (b) The gravitational field intensity at a point is defined as the force acting on a unit mass placed at that point in the field.

Thus, the gravitational field intensity is given by:

$$E = \frac{F}{m}$$

Now at distance x from the centre of Earth, the gravitational force is

$$\mathbf{F} = \frac{GMm}{x^2} \hat{x}$$

$$\mathbf{E} = \frac{\mathbf{F}}{m} = \frac{GMm}{x^2} \hat{x} = \frac{GM}{x^2} \hat{x}$$

$$|\mathbf{E}| = \frac{GM}{x^2}$$

or

$$\mathbf{E} = \frac{GM}{x^2}$$

On Earth

$$\mathbf{E} = \frac{\mathbf{F}}{m} = \frac{\text{Force}}{\text{mass}} = \text{acceleration}$$

$$\mathbf{E} = \frac{\mathbf{F}}{m} = g$$

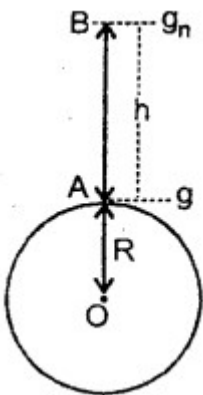
So, the intensity of the gravitational field at the surface of Earth is equal to the acceleration due to gravity.

3. Answer:

Let M , R be the mass and radius of the earth with centre O .

g = acceleration due to gravity at a point

At on Earth's surface.



(a) Variation of g with height: Let g_h be the acceleration due to gravity at a point B at a height h above the earth's surface

$$\therefore g = \frac{GM}{R^2} \quad \dots (1)$$

and
$$g_h = \frac{GM}{(R+h)^2} \quad \dots (2)$$

$$\frac{(2)}{(1)} \text{ gives, } \frac{g_h}{g} = \frac{R^2}{(R+h)^2} = \frac{1}{\left(1 + \frac{h}{R}\right)^2}$$

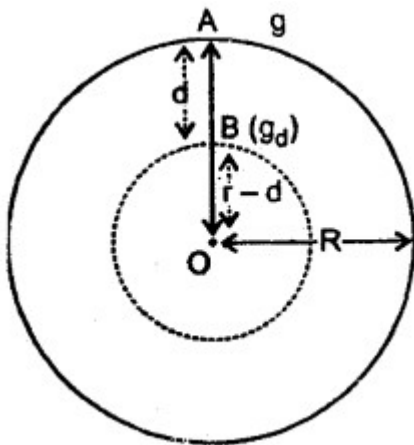
or
$$g_h = g \left(1 + \frac{h}{R}\right)^{-2}$$

If $h \ll R$, then using Binomial Expansion, we get

$$\begin{aligned} g_h &= g \left(1 - \frac{2h}{R}\right) \\ &= g - \frac{2gh}{R} \quad \dots (3) \end{aligned}$$

Thus, from Eqn. (3), we conclude that acceleration due to gravity decreases with height.

(b) With depth: Let the Earth be a uniform sphere.



Let g_d = acceleration due to gravity at a depth d below earth's surface i.e., at point B.

Let ρ = density of Earth of mass M .

$$\therefore g = \frac{GM}{R^2} \quad \dots (i)$$

where
$$M = \frac{4}{3}\pi R^3 \rho$$

Also, let M' = mass of Earth at a depth d , then

$$M' = \frac{4}{3}\pi(R-d)^3 \rho$$

$$\begin{aligned} \therefore g &= \frac{G}{R^2} \frac{4}{3}\pi R^3 \rho \\ &= \frac{4}{3}\pi G \rho R \end{aligned} \quad \dots (ii)$$

Similarly

$$\begin{aligned} g_d &= \frac{GM'}{(R-d)^2} \\ &= \frac{4}{3}\pi G \rho (R-d) \end{aligned} \quad \dots (iii)$$

$$\therefore \frac{g_d}{g} = \frac{R-d}{R} = 1 - \frac{d}{R}$$

or

$$g_d = g \left(1 - \frac{d}{R} \right) \quad \dots (iv)$$

From equation (iv), we see that acceleration due to gravity decreases with depth.

Special case: At the centre of Earth, $d = R$

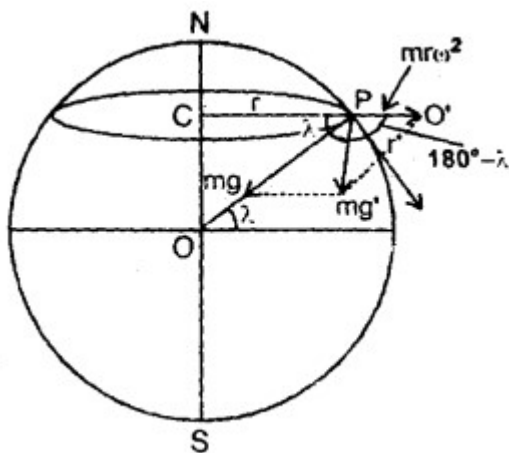
$$\therefore g_d = 0$$

Hence an object at the centre of Earth is in a state of weightlessness.

(c) Variation of g with latitude:

Let m = mass of a particle at a place P of latitude X .

ω = angular speed of Earth about axis NS .



As the earth rotates about the NS axis, the particle at P also rotates and describes a horizontal circle of radius r ,

where $r = PC = OP \cos \lambda, = R \cos \lambda$

Let g' be the acceleration due to gravity at P when the rotation of Earth is taken into account. Now due to the rotation of the earth, two forces that act on the particle at P are:

Its weight mg , acting along with PO.

Centrifugal force $m\omega^2 r$ along PO'.

$\therefore$ The angle between them $= 180^\circ - \lambda$

$\therefore$ According to the parallelogram law of vector addition

$$\begin{aligned} mg' &= \sqrt{(mg)^2 + (mr\omega^2)^2 + 2(mg)(mr\omega^2)\cos(180^\circ - \lambda)} \\ &= \sqrt{m^2g^2 + m^2r^2\omega^4 - 2m^2gr\omega^2\cos\lambda} \end{aligned}$$

$$\begin{aligned} \text{or } g' &= g\sqrt{1 + \frac{r^2\omega^4}{g^2} - \frac{2r\omega^2}{g}\cos\lambda} \\ &= g\sqrt{1 + \frac{\omega^4 R^2 \cos^2 \lambda}{g^2} - \frac{2R\omega^2}{g}\cos^2 \lambda} \\ &= g\sqrt{1 - \frac{2R\omega^2}{g}\cos^2 \lambda} \end{aligned}$$

[As $\frac{R\omega^2}{g}$ is very small ($= \frac{1}{289}$) so its square and higher powers are neglected.]

$$\therefore g' = \left(1 - 2\frac{R\omega^2}{g}\cos^2 \lambda\right)^{\frac{1}{2}}$$

Using binomial expansion, we get

$$\begin{aligned} g' &= \left(g - \frac{1}{2} \times \frac{2R\omega^2}{g} \times g \cos^2 \lambda\right) \\ g' &= g - R\omega^2 \cos^2 \lambda \end{aligned}$$

$\Rightarrow g$ decreases with the rotation of the earth.

At poles, $\lambda = 90^\circ$, $\therefore g' = g_p = g$

At equator, $\lambda = 0$, $g' = g_e = g - R\omega^2$.

Clearly $g_p > g_e$.

Assertion Reason Answer:

1. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

Explanation:

Because gravitational force is always attractive in nature, and everybody is bound by this gravitational force of attraction of earth.

2. (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion.

Explanation:

Both assertion and reason are true, but reason is not correct explanation of the assertion.

Case Study Questions-

1. If a stone is thrown by hand, we see it falls back to the earth. Of course using machines we can shoot an object with much greater speeds and with greater and greater initial speed, the object scales higher and higher heights. A natural query that arises in our mind is the following: can we throw an object with such high initial speeds that it does not fall back to the earth? Thus minimum speed required to throw object to infinity away from earth's gravitational field is called escape velocity.

$$v_e = \sqrt{2gr}$$

Where g is acceleration due to gravity and r is radius of earth and after solving v_e 11.2 km/s. This is called the escape speed, sometimes loosely called the escape velocity. This applies equally well to an object thrown from the surface of the moon with g replaced by the acceleration due to Moon's gravity on its surface and r replaced by the radius of the moon. Both are smaller than their values on earth and the escape speed for the moon turns out to be 2.3 km/s, about five times smaller. This is the reason that moon has no atmosphere. Gas molecules if formed on the surface of the moon having velocities larger than this will escape the gravitational pull of the moon. Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them. In particular, their orbits around the earth are circular or elliptic. Moon is the only natural satellite of the earth with a near circular orbit with a time period of approximately 27.3 days which is also roughly equal to the rotational period of the moon about its own axis.

- i. Time period of moon is
- 27.3 days
 - 20 days
 - 85 days
 - None of these
- ii. Escape velocity from earth is given by
- 20 km/s

- b. 11.2 km/s
 - c. 2 km/s
 - d. None of these
- iii. Define escape velocity. Give its formula
- iv. Why moon don't Have any atmosphere?
- v. What is satellite? Which law governs them?
2. Satellites in a circular orbits around the earth in the equatorial plane with $T = 24$ hours are called Geostationary Satellites. Clearly, since the earth rotates with the same period, the satellite would appear fixed from any point on earth. It takes very powerful rockets to throw up a satellite to such large heights above the earth but this has been done in view of the several benefits of many practical applications. Thus radio waves broadcast from an antenna can be received at points far away where the direct wave fails to reach on account of the curvature of the earth. Waves used in television broadcast or other forms of communication have much higher frequencies and thus cannot be received beyond the line of sight. A Geostationary satellite, appearing fixed above the broadcasting station can however receive these signals and broadcast them back to a wide area on earth. The INSAT group of satellites sent up by India is one such group of geostationary satellites widely used for telecommunications in India. Another class of satellites is called the Polar satellites. These are low altitude (500 to 800 km) satellites, but they go around the poles of the earth in a north-south direction whereas the earth rotates around its axis in an east-west direction. Since its time period is around 100 minutes it crosses any altitude many times a day. However, since its height h above the earth is about 500-800 km, a camera fixed on it can view only small strips of the earth in one orbit. Adjacent strips are viewed in the next orbit, so that in effect the whole earth can be viewed strip by strip during the entire day. These satellites can view polar and equatorial regions. at close distances with good resolution. Information gathered from such satellites is extremely useful for remote sensing, meterology as well as for environmental studies of the earth.
- i. Time period of geospatial satellite is
- a. 24 hours
 - b. 48 hours
 - c. 72 hours
 - d. None of these
- ii. Polar satellites are approximately revolving at height of
- a. 500 to 800km
 - b. 1500 to 2000 km
 - c. 3000 to 4000 km
 - d. None of these

- iii. Which satellite used to view polar and equatorial regions?
- iv. Write note on polar satellites
- v. Write a note on geostationary satellite. Give its applications.

Case Study Answer-

1. Answer

- i. (a) 27.3 days
- ii. (b) 500 to 800km
- iii. Polar satellites are used to view polar and equatorial regions as they rotate on poles of earth.
- iv. The escape speed for the moon turns out to be 2.3 km/s, about five times smaller than that of earth. Therefore all atmospheric gas can go easily out of atmosphere of moon. This is the reason that moon has no atmosphere.
- v. Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them.

2. Answer

- i. (a) 24 hours
- ii. (a) Pascal's law
- iii. Polar satellites are used to view polar and equatorial regions as they rotate on poles of earth.
- iv. Polar satellites are low altitude (500 to 800 km) satellites, but they go around the poles of the earth in a north-south direction. Since its time period is around 100 minutes it crosses any altitude many times a day. Information gathered from such satellites is extremely useful for remote sensing, meteorology as well as for environmental studies of the earth.
- v. Satellites in circular orbits around the earth in the equatorial plane with time period same as earth are called Geostationary Satellites.
Applications:- Radio waves broadcast. Satellites widely used for telecommunications in India. GPS system, navigation system , defence etc.